

Cracking the Supersolid

Philip Phillips

Department of Physics, University of Illinois 1110 W. Green Street, Urbana, IL 61801, U.S.A.

Alexander V. Balatsky

*Theoretical Division and Center for Integrated Nanotechnology,
Los Alamos National Laboratory, Los Alamos, NM 87245, USA*

PACS numbers:

We routinely teach students in a first course in physics that the rotational motion of a rigid body is strongly determined by its moment of inertia. Such an exercise would be futile in accounting for the rotational properties of a supersolid¹ in which some of the atoms remain still while the rest rotate with the container. A supersolid is one of the truly enigmatic quantum states of matter, whereby the very same atoms exhibit simultaneously crystalline order and a superfluid stiffness. Indeed, it is the richness of quantum mechanics that permits this seemingly paradoxical behaviour. A series of torsional oscillator experiments on solid He⁴ report that anywhere from .14 to 20%^{2,3,4,5,6,7} of the atoms remain still while the rest rotate with the container. The current debate regarding the observation of a missing moment of inertia (MMI) in solid He⁴ centers on whether superflow and hence supersolidity is the root cause or whether some other perhaps non-equilibrium phase, such as a glass (which may or may not support a superfluid stiffness), might be operative.

Of course, non-controversial instances of a missing moment of inertia or non-classical rotational inertia exist. Consider a rotating container filled with liquid He⁴. At sufficiently low temperature, the angular momentum of the liquid He⁴ disappears provided the container rotates sufficiently slowly so that vortices are not excited. This effect, first observed by Hess and Fairbank⁸ is the analogue of the Meissner effect in a superconductor and represents the benchmark test of equilibrium superfluidity. MMI in a superfluid requires that both the single-atom wavefunctions extend over the entire sample and some fraction of atoms Bose condense. For a perfect crystalline solid, that is, one in which the number of atoms equals the number of lattice sites, neither of these conditions can be satisfied.

To rectify crystallinity with a MMI, Andreev and Lifshitz⁹ and others^{1,10} focused on the quantum mechanical motion associated with vacancy or interstitial defects. Such defects occur, in principle, in any solid and give rise to incommensuration. Since they are bosons, vacancies or interstitials can Bose condense at sufficiently low temperature. In such a scenario, the defects are the superfluid while the He⁴ atoms maintain long-range crystal order.

Although the vacancy scenario has been adopted¹¹ to explain the MMI in the 2004 torsional oscillator experiments of Kim and Chan² (KC) on solid He⁴, such ex-

planations ultimately leave a residue. First, accurate Monte Carlo calculations indicate that vacancy-type defects phase separate in pure solid He⁴ rather than form a supersolid^{12,13}. Second, the experimental bounds on the number of vacancy/interstitial defects determined by Simmons, et al.¹⁴ is far lower (0.3%) than the modest superfluid fraction of 2% seen in the early experiments of Kim and Chan. In fact, based on the expression for the Bose condensation temperature in the dilute regime, it is straightforward to show that vacancy defects with a density of .3% already condense at 200mK, the onset temperature for the MMI in the KC experiments, a further indication that the defect scenario cannot quantitatively explain the data. Third, Blackburn, et al.¹⁵ have observed no low-temperature anomaly in the Debye-Waller factor below the onset temperature of the MMI in the KC experiment. Absence of such an anomaly casts doubt on the MMI of KC being associated with a true phase transition driven by defects. Fourth, Rittner and Reppy^{7,16} have demonstrated that the MMI signal is acutely sensitive to the quench time for solidifying the liquid. In one extreme, they found the MMI to be absent in fully annealed samples. This experiment is still in dispute as not all groups^{17,18,19} have been able to eliminate the MMI by sample annealing. In the other, the MMI increased to an astounding 20% (see Fig. 1) in samples in which the solidification from the liquid occurred in less than 2 minutes⁷. A narrow annular region which maximized the surface to volume ratio enabled such rapid cooling of the sample and ensured a high degree of frozen-in disorder. This striking feature suggests that supersolidity is not intrinsic to pure solid He⁴. Rather, some kind of disorder, dislocation-induced plasticity, or glassy ordering^{20,21,22} is the efficient cause.

Ultimately, the sharp test of supersolidity is persistent mass flow, much the way a persistent current obtains in a superconductor. The key success thus far in this regard is that of Sasaki et al.²³ who observed mass flow only in samples containing grain boundaries. However, the precise relationship between this experiment and the torsional oscillator measurements is unclear because mass flow was observed at temperatures (1.1K which is not far from the bulk superfluid transition temperature) vastly exceeding the onset temperature for MMI in the torsional oscillator experiments⁵, namely $T_c = 0.2K$. The team led by Beamish²⁴ has looked specifically for pressure-induced

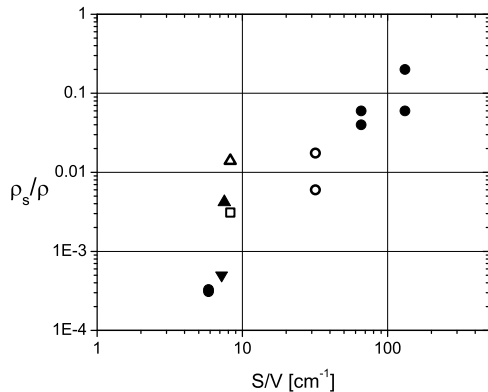


FIG. 1: Rittner and Reppy⁷ noticed that all of the experimental values for the MMI fall onto a universal curve as a function of the surface to volume ratio.

mass flow through two parts of the sample separated by a set of micron-size capillaries and have seen no tell-tale signature. Relying on the fact that true superflow should exhibit a thermodynamic signature, in particular a diminished entropy at T_c , Todoshchenko²⁵ et al. measured the melting curve of He⁴ between 10 and 320mK with an accuracy of $0.5\mu\text{bar}$. They observed no deviation from the expected T^4 law due to phonons in ultra-pure samples with a He³ concentration of 0.3ppb. Such an experiment is not sufficient to rule out superflow in the highly polycrystalline samples but is a clear indication that superflow is not intrinsic to the pure hcp He⁴ but an extrinsic disorder-driven effect.

Hence, the question remains: Is the observation of MMI in the torsional oscillator experiments now seen by numerous groups around the world an example of true superflow? Short of a direct observation of persistent mass flow, it is essential that thermodynamic measurements of the kind performed by Todoshchenko²⁵ be carried out on the polycrystalline samples of Rittner and Reppy⁷. In addition, neutron scattering and x-ray tomography measurements below 200mK on the polycrystalline samples could offer unprecedented insight into the defect structure that enables the observed MMI.

Ultimately, if disorder is key, then several questions arise. Most notably, why is the signal in vycor so anomalously low? Vycor has a surface to volume ratio that exceeds any of the samples used in Fig. 1 by four orders of magnitude, while its MMI is only 0.4%. Why? Can grain boundaries account for a 20% MMI? (see Fig.1). Can a system in which 20% of the atoms flow along grain boundaries be properly thought of as a crystal? What is the quantitative theory behind the He³ enhancement of T_c ?

In light of the disorder data, there are two classes of theoretical proposals left standing. First, are the theories^{20,21} that rely on some sort of disordered supercomponent present in solid He⁴. The essential physics of these approaches is captured by the disordered Bose-Hubbard model²⁶. In fact, the only calculations²¹ that provide a quantitative explanation of the enhancement of T_c caused by He³ impurities are based on this model. Within such a model, one can argue²¹ that the absence of MMI in the pure system arises because in the clean system, He⁴ atoms in an hcp lattice form a Mott insulator. That is, the atoms themselves are the lattice sites for the Mott insulator. Disorder^{21,26} is expected to destroy the Mott state and give rise to a superfluid. The precise mechanism by which this occurs, via mid-gap states or self-doping, is still unclear. Second, a number of proposals have been offered for bosonic glassy states in which MMI is obtained without ever invoking superflow²². Torsional oscillator experiments ultimately measure the changes in mechanical properties, oscillation period, and damping. The connection to MMI is indirect and is not interpretation free. One common feature of any kind of normal glassy state would be that the mechanical response of the solid and hence the torsional oscillator properties are changing at the onset of the glass state. Glassy state proposals also predict a frequency-dependent decay of the oscillation amplitude. The challenge for theory and experiment would be to characterize bulk He⁴ samples with enough precision to rule in or out such non-superflow scenarios.

Fig. 1 and the dramatic enhancement He³ has on T_c lay plain that the standard textbook supersolid falls short as an adequate explanation of the experiments. What is clear is that the true answer is hidden in the disorder.

¹ A. J. Leggett, Phys. Rev. Lett. **25**, 2543 (1970).

² E. Kim and M.H.W. Chan, Nature (London) **427**, 225 (2004).

³ E. Kim and M.H.W. Chan, Science **305**, 1941 (2005).

⁴ A. C. Clark and M.H.W. Chan, J. Low Temp. Phys. **138**, 853 (2005).

⁵ E. Kim and M.H.W. Chan, Phys. Rev. Lett. **97**, 115302 (2006).

⁶ M. Kondo, et al., cond-mat/0607032.

⁷ A. S. C. Rittner and J. D. Reppy, cond-mat/0702665.

⁸ G. B. Hess and W. M. Fairbank, Phys. Rev. Lett. **19**, 216 (1967).

⁹ A.F. Andreev and I.M. Lifshitz, Sov. Phys. JETP **29**, 1107 (1969).

¹⁰ G. V. Chester and L. Reatto, Phys. Rev. **155**, 88 (1967).

¹¹ P.W. Anderson, W.F. Brinkman and D.A. Huse, Science **310**, 1164 (2005).

¹² M. Boninsegni, Phys. Rev. Lett. **97**, 80401 (2006).

- ¹³ D. Ceperley, private comm.
- ¹⁴ S. M. Heald, D. R. Baer, and R. O. Simmons, Phys. Rev. B **30**, 2531 (1984).
- ¹⁵ E. Blackburn, et al. cond-mat/0702537.
- ¹⁶ A.S. Rittner and J.D. Reppy, Phys. Rev. Lett. **97**, 165301 (2006).
- ¹⁷ E. Kim and M. H. W. Chan, Phys. Rev. Lett. **97**, 115302 (2006).
- ¹⁸ A. Penzev, et al., cond-mat/0702632.
- ¹⁹ K. Shirahama, et al., Bull. Am. Phys. Soc. **51**, 450 (2006).
- ²⁰ M. Boninsegni, N. Prokof'ev, and B. Svistunov, Phys. Rev. Lett. **96**, 105301 (2006).
- ²¹ J. Wu and P. Phillips, cond-mat/0612505.
- ²² Z. Nussinov, A.V. Balatsky, M.J. Graf, and S.A. Trugman, cond-mat/0610743, submitted to Phys. Rev. B; A.V. Balatsky, Z. Nussinov, M. Graf and S. Trugman, Phys. Rev. B **75**, 094201, (2007).
- ²³ S. Sasaki, R. Ishiguro, F. Caupin, H.J. Maris, and S. Balibar, Science **313**, 1098 (2006).
- ²⁴ J. Day and J. Beamish, Phys. Rev. Lett. **96**, 105304, (2006)
- ²⁵ I. A. Todoshchenko, H. Alles, H. J. Junes, A. Ya. Parshin, and V. Tsepelin, cond-mat/0703743.
- ²⁶ M.P.A. Fisher, et. al., Phys. Rev. B **40**, 546, (1989).